Finding Unit and Last two digits of an expression

To find the last digit of an expression which is in the format of a^b

Cyclicity of unit digit of an expression:

	Power			
Base	1	2	3	4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6
4	4	6		
9	9	1		

From the table it is clearly visible that for all the numbers whose unit digit in the format of 2^b, the unit digits are respectively $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 6$, $2^5 = 2$

Similarly we can find unit digits for the remaining numbers easily.

Please observe, The cyclicity of the numbers 2, 3, 7, 8 is 4, and for 4, 9 is 2 as the pattern is repeating after power 4. The cyclicity of 0, 1, 5, 6 is 1.

Problem 1:

What is the unit digit of the expression 317^{171}

Solution:

Here we can concentrate only on the unit digit of the base and the power. Unit digit of the base is 7 so from the table its cyclicity is 4.
Let us find the remainder when 171 is divided by 4. For the divisibility rule for the 4 is to find the remainder of the last two digits of 171, so 71 when divided by 4 gives a remainder 3. So from the table unit digit of 7^3 is 3.

Problem 2:

Find the unit digit of the expression $1^{781}+2^{781}+3^{781}+4^{781}\ldots\ldots\ldots\ldots\ldots\ldots\ldots+9^{781}$

Solution:

we know that 781 when divided by 4 gives a remainder 1. As is visible clearly from the table that for every unit digit after the power 4 the same unit digit repeats.

So unit digit = 1 + 2 + 3 ....9 = 45 so unit digit is 5

Last two digits of an expression:

If we need to find the last two digits of an expression we need to consider the last two digits of the base. We need to consider two cases separately.

Case 1: Numbers which base end with 1.

These numbers are in the format of $\ldots abc1^{\ldots xyz}$.

Unit digit of this expression is always 1 as the base ends with 1. For the tenth place digit we need to multiply the digit in the tenth place of the base and unit digit of the power and take its unit digit

Example: the last two digits of 2341^{369} = (4 x 9),1 = 61

Case 2: Numbers which end with 5 as unit digit

The last two digits are always 25 or 75. Let the give number is $..ab5^{xyz}$. If the product of units digit of the power (i.e., z) and digit left to the 5 in the base (i.e.,b), is even then last two digits of the expression is 25, If the power is odd then it is 75.

Example: last two digits of 2345^{369} are 25 as the product 4*9 = 36 which is even.

Case 3: Numbers which end with 3, 7, 9.

we need to change the unit digits 3, 7, 9 to 1 by little modification. From the unit digit table we can find that 3, 7, 9 may give unit digit 1 for the powers of 4, 4, 2 respectively.

In finding the last two digits of an expression 2343^{4747} we can re-write the expression 43^{4747} as we are concerned with only last two digits only.

Now we consider $(43^4)^{1186}.43^3 \Rightarrow (43^2\times 43^2)^{1186}.43^3$

$43^2 = 1849$ so

We have to consider the last two digits of 1849 $\Rightarrow (49\times 49)^{1186}.43^3$.

Now49 x 49 will give unit digits as 01, $\Rightarrow (01)^{1186}.43^3$

$\Rightarrow 01 \times 07 \Rightarrow 07$.

Case 4: Numbers which end with 2, 4, 6.

Firstly we should by-heart these two rules: 2^{10} raised to the even power always give last two digits as 76, and when raised to the odd power give the last two digits as 24.

Then we should take 2's separately from the given number and we need to apply the above rule.

Example: Find the last two digits of 48^{199}

By re-writing 48^{199} = $(2^4 \times 3)^{199}$ = $2^{796} \times 3^{199}$ = $2^{10\times79+6} \times 3^{199}$ = $(2^{10})^{79}.2^6.3^{199}$

Now 2^{10} raised to an odd power gives 24 as the last two digits. So

$(24).2^6.3^{199} \Rightarrow 24 \times 64 \times (3^{4\times49+3}) \Rightarrow 24 \times 64 \times 81^{49} \times 3^3 \Rightarrow 24 \times 64 \times 21 \times 27$ = 12